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THE GREAT L'S OF FRENCH MATHEMATICS

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Abstract

During and surrounding the 18th century, France was the dominant player in the world of mathematics. This article highlights the lives and accomplishments, of 11 of the great French mathematicians at this time, who all had an interesting characteristic in common.

1. Introduction

The history of mathematics dates back a long time, and surely enjoys some periods of remarkable achievements. The early Greek era involving Archimedes, Euclid, Diophantus, and Pythagoras really got the mathematical ball rolling. Periods of lackluster to minimal growth soon followed, but then mathematics received a major boost when it landed in Italy in the 15th-16th century for its developments in algebra. Within the next 200 years, mathematics took off like it was shot out of a cannon. Outside of the English genius Isaac Newton and the Swiss Leonhard Euler, a large majority of the growth was centered in mainland Europe, specifically Germany and France. The Germans included such greats as Carl Gauss,

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Karl Weierstrass, Friedrich Wilhelm Bessel, Carl Gustav Jacobi, Peter Gustav Dirichlet, Christoph Gudermann, and Ernst Kummer, while the French were comprised of a number of individuals, who all shared a common and unique characteristic, namely, their last names began with the same letter, L. In this article, a brief mathematical summary is presented of each of messieurs Lacroix, Lagrange, Laguerre, Lamé, Laplace, Lebesgue, Leclerc, Legendre, L'Hôpital, Liouville, and Lucas arranged in chronological order.

2. Guillaume François Antoine Marquis de L'Hôpital (1661-1704)

Guillaume de L'Hôpital represents the first of these great mathematicians in that he was the earliest born (in Paris). A couple of other distinguishing features about him include that he probably had the longest name (what is shown here is not all of it) and was born in the wealthiest family [45]. L'Hôpital's family had been a prominent one in France for a good 400-500 years, and, in fact, his father was a lieutenant-general in the King's army, while his father-in-law also served in the army.

While in school L'Hôpital developed strong interests in mathematics, clearly his favourite subject. But as soon as he could, he followed family tradition and began a career in the military. He served as a captain in a cavalry regiment for a while, but then he resigned from the army because of poor eyesight, although some believe it was his passion for mathematics that led him to abandon a military career.

L'Hôpital was actually a very capable mathematician, and had published a few brief mathematical notes, but nothing noteworthy enough to place him in the history books. In the latter part of 1691, he had the good fortune to make the acquaintance of the 24 years old Swiss Johann Bernoulli, the younger brother of Jacob. Bernoulli had been travelling through Europe giving lectures on the latest developments in mathematics, particularly, the differential and integral calculus that had been developed by the German genius Gottfried Leibniz. L'Hôpital proposed to hire Johann (with a retainer of 300 francs) to serve as his special mathematics tutor. One special stipulation in this proposal was that Bernoulli must share with L'Hôpital, and no one else, any mathematical discovery that he made [14]. Bernoulli rapidly accepted this proposition although it was not obvious whether he did it for the monetary gain or the increased publicity and social status of being associated with an important French nobleman.

The correspondence between the two lasted for about a year until late in 1695 when Bernoulli moved on to other activities. But then in 1696, L'Hôpital published his famous book "*Analyse des Infiniments Petits Pour L'intelligence des Lignes Courbes*" (Infinitesimal calculus with applications to curved lines). This was the first textbook to be written on the differential calculus, with no mention of the integral calculus [45]. L'Hôpital gave some credit in the book to both Johann and Jacob Bernoulli, and of course to Leibniz and Newton. It is in Chapter 9 of this book, where the all-important L'Hôpital's rule, that all calculus students of today know about, is introduced and which virtually cemented L'Hôpital's name in mathematical fame [60]. This is the rule for finding the limit of a rational function, whose numerator and denominator functions tend to zero (or infinity) at a point. It is debatable, though, whether this should more properly be called *Bernoulli's rule*.

One other highlight of L'Hôpital's life that occurred around this time involved the famous *brachistochrone problem* posed by Johann Bernoulli in the German journal *Acta Eruditorum* in 1696 [42]. Bernoulli proposed the following to all the mathematicians in the world.

Given two points A and B in a vertical plane, what is the curve traced out by a point acted on only by gravity, which starts at A and reaches B in the shortest time?

This problem actually dated back 60 years earlier to Galileo, who thought that he knew the solution would involve a circular arc, but he was incorrect. Bernoulli's posing of the problem produced five correct solutions, those by both Johann and brother Jacob, Newton, Leibniz, and L'Hôpital. This put L'Hôpital in pretty special company. The solutions by both Bernoullis, Newton, and Leibniz were all published in the May, 1697 issue of *Acta*, while L'Hôpital's did not appear until roughly 300 years later.

L'Hôpital was soon (in 1699) to be given honorary status in the prestigious *Académie des Sciences* because of his mathematical work and not his family name. This organization, recently founded in Paris in 1666 by Jean-Baptiste Colbert, was reorganized in 1699 under the royal patronage of Louis XIV, and was seen as a forum for the development of science. Memberships were divided into subject disciplines, of which there were two main categories, namely, mathematical sciences and physical sciences. L'Hôpital fit nicely into the former.

3. Georges-Louis Leclerc (1707-1788)

Georges Leclerc was born in a wealthy family in 1707 at Montbard, located just a short distance southwest of Paris. When Leclerc was ten years old his mother inherited a large sum of money, which allowed his father to become lord of Montbard and Buffon (an estate that his mother also inherited). By the age of 20, Leclerc had added the suffix *de Buffon* to his name, and that is predominately how he has come to be known in the literature.

He entered the *Jésuite Collège des Godrans* in Dijon around 1717 and, even though three of his brothers went on to join the Church. Leclerc's father really wanted him to study law. This did not exactly pan out because Leclerc was more interested in mathematics [43].

In 1727, Buffon made an important mathematical discovery on his own, namely, the expansion known as the *binomial theorem*, which gives the expansion of $(a + b)^n$ [4]. This result was not new, it had been known for a long time, and Newton had generalized it some 50 years earlier. Sometime during the period of 1727-1730, Buffon made the acquaintance of the Swiss mathematician Gabriel Cramer. Recently appointed to the Geneva Academy, Cramer was travelling throughout Europe getting acquainted with leading mathematicians when he met Buffon in Paris. This was a fortunate event in Buffon's life because he and Cramer would correspond on mathematics for a number of years, covering topics on mechanics, geometry, probability, number theory, and the calculus. Cramer was particularly good at publishing and editing mathematics treatises. He published, for example, the *Complete Works* of Johann Bernoulli (at Johann's request), the *Works* of Jacob Bernoulli, and the correspondence between Johann Bernoulli and Gottfried Leibniz [44]. Cramer's name lives in many high school algebra classes, where *Cramer's rule* is a common method for solving a system of linear equations by using determinants.

Buffon got his advanced education at both the University of Dijon and Angers University (several hundred miles southwest of Paris). Here he concentrated on mathematics and science but also he had interests in medicine and botany. In 1730, while a student at Angers, he became involved in a duel and as a result had to flee the town [33].

Being wealthy as he was, Buffon was able to make acquaintances with people in the highest ranks of the political and scientific circles in Paris. The minister of the French navy, in particular, wanted to improve the construction of ships at war, so he asked Buffon to study the tensile strengths of various timber. In 1733, Buffon published his first mathematical work, *Mémoire sur le jeu de Franc-Carreau*, which introduced differential and integral calculus into probability theory. This material would set the stage for what would soon be Buffon's defining moment in mathematics. The publication was so well received that Buffon was elected to the Royal Academy of Sciences in 1734.

Buffon was to spend the rest of his life dividing time between mathematics, botany, forestry, and natural history. Because of this division of time, Buffon did not achieve the same mathematical rank as the other great French mathematicians in this paper. One mathematical contribution of Buffon's during this time was his published translation of Newton's *method of fluxions and infinite series*, another work having to do with the differential calculus.

In 1739, Buffon was appointed as keeper of the royal botanical garden, the *Jardin du Roi*. He held this position to the end of his life, and was instrumental in transforming the garden into a major research center and museum [43].

Outside of mathematics, Buffon is best remembered for his 1749 publications of his natural history works *Histoire Naturelle, Générale et Particulière*, and *Théorie de la terre*. Buffon believed that everything developed through natural phenomena. He proposed a method of creation of the planet, which involved the collision of a comet with the sun. Similarly, he believed that life on earth came about through the appearance of organic matter. Buffon published *Les Époques de la Nature* in 1788, the year of his death, where he postulated that the planet was approximately 75,000 years old, much older than the 6,000 years proclaimed by the Church [5]. Except for Aristotle, Mendel, and Darwin, not many have had as far-reaching an influence on the study of life as Buffon. He definitely brought the idea of evolution into the realm of science. Though undoubtedly among the greatest scientists of his time, Buffon was not held in high regard among his peers because he continually challenged the authority of biologists, chemists, other mathematicians, and the theologians.

It is in the year 1777, though, when Buffon really made his name in the mathematics world. The science of probability blossomed in the 20th century although its history dates back to the 16th century when Gerolamo Cardano published his book *Liber de Ludo Aleae* (the book of games and chance) [2]. Buffon, who had interests in both probability and calculus, proposed the following problem, known as *Buffon's needle problem*.

"Let a needle of length L be thrown at random onto a horizontal plane ruled with parallel lines spaced by a distance d apart $(d \geq L)$ from each other. What is the probability that the needle will intersect one of the lines?"

Using the techniques and tools of the newly created differential and integral calculus, Buffon was able to show that this probability was exactly P, where [2]

$$P = \frac{\frac{1}{2} L \int_0^{\pi} \sin \phi \, d\phi}{\frac{\pi d}{2}} = \frac{2L}{\pi d}.$$

One astonishing result here is the appearance of the constant π. Since π commonly appears in problems involving circles (since π is defined as the ratio of a circle's circumference to its diameter), it does not seem logical that it should appear in this setting – but it does. In particular then, if L is chosen equal to d, the probability of the needle crossing some line is equal to $\frac{2}{\pi}$, or roughly 0.64. By performing this experiment, a large number of times (known as a *Monte Carlo method*), one can compute an approximate value for π. Computing a decimal approximation for π has been a favourite diversion for mathematicians for the past 2000 years – since the days of Archimedes.

When Buffon was 76 years old, he was made *Comte de Buffon*. He is remembered by having both a lunar crater and a Parisian street named after him.

4. Joseph-Louis Lagrange (1736-1813)

Joseph Lagrange is different from all the other French mathematicians in this article because he was born outside of France (Turin, Italy), and was baptized with the name Giuseppe Lodovico Lagrangia [47]. His parents and ancestors were of mixed Italian and French blood, with the French predominating. Lagrange was the youngest of 11 children, and in fact, the only one to survive into adulthood. Initially, Lagrange's parents were fairly wealthy, but later Lagrange's father lost most of his money via poor speculation. Later in the life, Lagrange would comment on this unfortunate happening by saying "If I had been rich, I probably would not have devoted myself to mathematics" [47].

And boy did Lagrange devote himself to mathematics. So much so that he was acclaimed as the pre-eminent mathematician in all of France during the 18th century, and together with the Swiss genius Euler, the two of them pretty much stood a top all of Europe. Napoléon thought so much of him that he stated "Lagrange is the lofty pyramid of the mathematical sciences", and then proceeded to bestow him with honours such as being a Senator and a Count of the Empire [3].

As for Lagrange's magnificent career, it could most sensibly be partitioned into three classes. The first would be the 30 years spent in Italy, followed by the 21 years in Berlin, and finally his last 26 years in Paris.

While attending the *Collège of Turin* with the intent of becoming a lawyer, he soon became infatuated with physics and mathematics, in particular, with geometry and analysis. He published his first mathematical paper in 1754, which drew an analogy between the binomial theorem and the derivatives of the product of functions

$$\frac{d^n}{dx^n}(fg).$$

It was during this time that Lagrange began communicating with Euler, something he would do many times. Later in the same year, he made some discoveries of the *tautochrone* problem, which led him to further contributions of the new field of the calculus of variations. By now Lagrange had really decided to be a mathematical analyst and not a geometer. This analytical preference was most evident in his masterpiece, the *Mécanique analytique* (analytical mechanics), which he wrote in 1754, but did not publish until 1788, in Paris [7]. The following year, while only 19 years old, Lagrange was appointed professor of mathematics at the Royal Artillery School of Turin, and the next year (1756) was elected to the Berlin Academy, thanks to the efforts of Euler.

During the years from 1759 to 1773, he contributed heavily to the first four volumes of the journal *Mélanges de Turin* with papers, which covered such topics as the propagation of sound, vibrating strings, fluid mechanics, planetary orbits, and systems of differential equations [47].

In 1766, Lagrange accepted Frederick II's offer to come to Berlin, where he succeeded Euler as Director of Mathematics at the Berlin Academy of Science. No doubt this offer was made to Lagrange because he had just won the top prize (on the movements of the four known satellites of Jupiter) from the Paris *Académie des Sciences* and Euler had just agreed to move to St. Petersburg. Lagrange was to win this top prize three more times, in 1772, 1774, and 1780. He was highly productive in a number of areas during his time in Berlin. In analysis, for example, if a function f has enough derivatives on an interval I, and $x, a \in I$ with $x \neq a$, then $f(x)$ can be approximated by a Taylor polynomial of degree n with a remainder (*Lagrangian form of the remainder*) given by [14]

$$\frac{f^{(n)}(\zeta_n)(x-a)^n}{n!}.$$

In abstract algebra, *Lagrange's theorem* says that if H is a subgroup of a finite group G, then the order of H divides the order of G [16]. In number theory, there is the sequence of *Lagrange numbers*, which appear as upper bounds on rational approximations to the continued fraction expansion of any real number [9]. Lagrange also gave the first published proof in number theory that any positive integer can be expressed as a sum of four squares of nonnegative integers $(\text{e.g., } 19 = 3^2 + 3^2 + 1^2 + 0^2)$ [36]. He was also the first to prove Wilson's conjecture that a positive integer n is prime, if and only if $(n-1)!+1$ is divisible by n [6]. Calculus students are familiar with the *method of Lagrange multipliers* used for optimizing functions of several variables subject to some constraints [60]. In numerical analysis, we have the *Lagrange interpolating polynomial*, which shows how to construct a polynomial of degree n that passes through any $n+1$ points, providing no two of them have the same first coordinate [21].

After the death of Frederick the Great, in 1786, Lagrange thought it best to leave Berlin, so he accepted Louis XVI's offer to come to Paris and be a member of the French *Académie des Sciences* [6]. This offer was especially attractive since Lagrange was not required to do any teaching. By now, though, Lagrange was slowing down and was not nearly as mathematically productive as he had been. His interests, instead, became metaphysics, human thought, the history of religions, languages, medicine, botany, and chemistry. He became good friends with the famed chemist Antoine Lavoisier, and was highly upset when Lavoisier was sent in 1794 to the guillotine during the Reign of Terror. He bitterly remarked, “It took them only a moment to cause his head to fall, and a hundred years perhaps will not suffice to produce its like”. Coincidentally, it was Lavoisier, who had saved Lagrange from arrest.

The *École Polytechnique* was founded in 1794 and Lagrange was its first professor of analysis. In 1795, the *École Normale* was founded, with the aim of training school teachers, and Lagrange taught classes there, although he was not an especially good teacher. Two years later, in 1797, he published *Théorie des fonctions analytiques*, the first work on the theory of functions of a real variable. Three years later, in 1800, his second work *Leçons sur le calcul des fonctions* on this topic appeared. In 1813, Napoléon named him *Grand croix de l'Ordre Impérial de la Réunion* (Grand Officer of the Legion of Honour) but, unfortunately, Lagrange died a week later.

5. Pierre-Simon Laplace (1749-1827)

Pierre-Simon Laplace was born in Beaumont-en-Auge, near Normandy on the northwest coast of France. His parents were hard working (but not wealthy) peasants, who worked on the land, for his father dabbled in the cider trade and his mother came from a fairly prosperous farming family. Not enough is known about Laplace's youth, since he was rather ashamed of his humble parents and rarely was inclined to talk about his childhood days [3]. He did spend his early days in a Benedictine school, and since his father wanted him to make a career of the Church, then he entered nearby Caen University, in 1765, to study theology. It so happens that the university was destroyed in 1944 by bombs during the war, but it was rebuilt in 1957. Within the next two years, Laplace discovered his mathematical talents and interests, and intending to pursue that for the rest of his life, he headed off to Paris to strike up an acquaintance with Jean le Rond d'Alembert. D'Alembert was a famous French mathematician, who together with the great Lagrange had deduced a fundamental classical law of motion.

A short while later, with D'Alembert's assistance, Laplace was appointed professor of mathematics at the Military School of Paris, *École Militaire* [3]. For the next half -century, Laplace's work would place him in elite status among the world's greatest scientists. He would do monumental work in the areas of astronomy, mathematical physics, and the theory of probability. As a mathematical astronomer, Laplace has been referred to as the "Newton of France". But on a darker side, his character has been blemished with his many acts of arrogance, snobbery, and heightened egotism. In addition, he demonstrated his "sunshine patriot" behaviour many times by continually shifting allegiance to whichever political faction would suit his favour best [61].

At the age of 21, he orally presented several mathematics papers to the illustrious *Académie des Sciences* in Paris. The first paper was on critical points of curves, and the second was on difference equations. The following year, 1771, saw his first publication, on the integral calculus, published in the prestigious German journal *Acta Eruditorum* [50]. Through his work in mathematical physics, Laplace was led to do extensive work in differential equations. Every standard textbook today on differential equations contains a chapter on the *Laplace transform*, which is an operator, useful in solving a particular class of equations and boundary value problems [26]. During his work with potential theory in physics, Laplace showed that the potential function V, if derivable from a conservative force, always satisfies the partial differential equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0,$$

which is known as *Laplace's equation* [61]. In mathematics and physics, the symbol Δ, known as *del*, is used to represent the *Laplace operator*, and represents [18]

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

Several other minor mathematical discoveries of Laplace are his method of solving equations of the second, third, and fourth order, and his establishment of the expansion theorem, the *Laplace expansion*, of determinants, which is more commonly known today as the *cofactor expansion* [31].

One of Laplace's great works was his five volumes of *Mécanique Céleste* (Celestial Mechanics), which was begun in 1799 and was finished in 1825 [7]. These volumes contain detailed information on the movement of the solar system, along with general laws of equilibrium and motions of solids (planets, moon, and comets) and also fluids and tides. In addition to these matters involving the solar system, Laplace worked extensively on other physical problems such as capillary action, double refraction, the velocity of sound, the existence of black holes and gravitational collapse, and the theory of heat. In accordance with this latter field, Laplace joined forces in 1780 with the renowned chemist Antoine Lavoisier to establish some results involving respiration and combustion. When the violent Reign of Terror consumed France in 1793-94, Lavoisier was one of the unfortunates (along with Marie Antoinette and Louis XVI) to feel the wrath of the guillotine (aptly called the "*National Razor*") [58]. Laplace was able to avoid the "Razor" because he volunteered to help calculate trajectories for the artillery and helped manufacture saltpeter for gunpowder.

Another classic work by Laplace was his *Théorie Analytique des Probabilités*, published in 1812. This treatise contains his definition of probability, Bayes' rule, remarks on mathematical expectation, and generating functions. It also gives methods for finding probabilities of compound events, a discussion of the method of least squares (linear regression), applications to mortality and life expectancy, and the mathematics behind the famous *Buffon needle problem*.

Laplace was, of course, elected to the *Académie des Sciences*, and in 1784, he was appointed as examiner at the Royal Artillery Corps. The following year he not only examined, but passed, a 16 years old candidate named Napoléon Bonaparte [50]. This was an association to have lasting effects, for Napoléon not only appointed Laplace as Minister of the Interior in 1799, but also awarded him the Legion of Honour in 1805, and in 1806 made him a Count of the First French Empire. When Napoléon fell from power, Laplace had no problem in transferring his loyalty to Louis XVIII, who not only appointed him a Marquis in 1817, but also made him President of the Committee to reorganize the *École Polytechnique* [16].

Laplace has been immortalized in France by having both a Paris street, an asteroid, and a landmark on the moon named after him. He is one of a small handful of people to have their name engraved on the Eiffel Tower. The European Space Agency's working-title for the International Europe Jupiter System Mission is "Laplace".

6. Adrien-Marie Legendre (1752-1833)

Adrien Legendre was either born in Paris (that is the majority opinion) or in southern France in Toulouse (by some accounts). In either case, most of his youth and teen years were spent in Paris, where he completed his schooling in 1770 at the *Collège Mazarin*. Here he received top quality education in both mathematics and physics. Since he came from a wealthy family, so there was no problem to seeking out the best education.

Legendre apparently showed a lot of mathematical promise, because in 1775, he was appointed (by D'Alembert) to teach at *École Militaire* in Paris and, in fact, to teach alongside Laplace [52]. He left teaching in 1780 to devote more time to research. In 1782, he won the top prize of the Berlin Academy for his description of the curve traced out by projectiles (such as cannon balls and bombs) subject to air resistance and different initial velocities [7]. One special benefit of winning this prize was that, it brought his name to the attention of the great Lagrange, who, in fact, was the Director of Mathematics at the Berlin Academy.

Legendre would engage in serious mathematical research for the next 50 years. His areas of study would include analysis, elliptic functions, classical mechanics, statistics, geometry, geodesy, number theory, and compiling data for mathematical tables. He began by studying the attraction of ellipsoids, such as planetary bodies. This highly promising work got him elected to the *Académie des Sciences*. It is also where the *Legendre equation* appears, namely,

$$(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1) = 0.$$

When α is a noninteger, the solutions to the equation are the *Legendre functions* and are given as power series, and they represent the attraction of an ellipsoid at any exterior point. When α is an integer, say $\alpha = n$, we get the *Legendre polynomials* $P_n(x)$, which were mentioned prominently in his celestial mechanics paper *Recherches sur la figure des planètes* in 1784. These polynomials are fairly standard material in present day courses on differential equations. The first six polynomials, for instance, are

$$P_0(x) = 1, \qquad P_3(x) = \frac{1}{2}(5x^3 - 3x),$$

$$P_1(x) = x, \qquad P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3),$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1), \qquad P_5(x) = \frac{1}{8}(63x^5 - 70x^3 + 15x),$$

and all the polynomials satisfy the boundary conditions $P_n(1) = 1$, $P_n(-1) = (-1)^n$, and an orthogonality condition $\int_{-1}^{1} P_m(x)P_n(x)dx = 0$ when $m \neq n$. The polynomials, which are useful in mathematical physics, can be explicitly determined by a Rodrigues formula

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n.$$

The subject of elliptic functions is one with which Legendre's name is closely connected. He worked in this area for roughly 40 years, publishing many papers and memoirs. Legendre also published numerous works on the integral calculus, which included definite integrals, double integrals, Eulerian integrals, and the gamma and beta functions. His work titled *Traité des Fonctions Elliptiques* appeared in three volumes in 1825, 1826, and 1830.

In 1794, Legendre published *Éléments de géométrie*, which became a leading elementary text on geometry for well over 100 years [35]. Legendre's goal was to make a marked improvement on Euclid's *Elements*, which he did since his book was highly effective in both Europe and the United States. In Legendre's book, he not only gives a different proof of the irrationality of π, but also furnishes the first proof of the irrationality of π^2 [16]. Legendre spent 30 unsuccessful years trying to prove Euclid's parallel postulate as a theorem from the other assumptions of Euclidean geometry [15]. Shortly after his death, papers began to appear on the new area of non-Euclidean geometry.

Legendre had the misfortune to work in many areas allied to the great German Carl Gauss; most notably among these was number theory [61]. In 1797-98, Legendre published his first edition of *Théorie des nombres*, a two-volume 859 pages work, which was the first treatise devoted exclusively to number theory. Here he mentions the *law of quadratic reciprocity* for residues (his proof was incorrect – later to be established in 1801 by Gauss). Legendre did introduce a new symbol, still very much in use today, known as the *Legendre symbol* $\left(\frac{A}{p}\right)$, which has the value +1, −1, or 0 depending on whether A is a quadratic residue (mod p), a quadratic nonresidue (mod p) or is divisible by the odd prime p [18]. Using this symbol, the law of quadratic reciprocity can be expressed in a formula

$$\left(\frac{p}{q}\right) \cdot \left(\frac{q}{p}\right) = (-1)^{\frac{p-1}{2}\frac{q-1}{2}},$$

where p, q are odd primes. Legendre was thus the first person to state the law in this form that is in vogue today [64].

Legendre and Gauss both investigated the fascinating study of the distribution of primes and tried to describe the number-theoretic function $\pi(n)$, which counts exactly how many primes are in the interval $[2, n]$. Legendre stated in 1796 that $\pi(n)$ could be approximated by

$\frac{n}{\ln(n) - 1.08366}$, while Gauss gave an alternative and slightly better formula known as the *logarithmic integral*. But Legendre was correct in stating that $\lim_{n \to \infty} \frac{\pi(n)}{n / \ln n} = 1$ [9]. This has come to be known as the *prime number theorem*, but was not properly proved until 100 years later by Charles de la Vallée and Jacques Hadamard. In 1798, Legendre established the interesting number theory representation result that a positive integer n cannot be expressed as the sum of three squares, if and only if n is of the form $n = 4^k(8m + 7)$, where m and k are nonnegative integers (such as $n = 7$ or 60) [19].

In 1806, appeared Legendre's *Nouvelles Méthodes pour la determination des orbites des comètes*, which is memorable because it contains, for the first time, the *method of least squares*, as used in probability and linear regression. Although Legendre is given credit for first formally proposing and mentioning it, both Gauss and Laplace are credited with developing the theory, the algorithm, and the mathematical foundation of the process.

Apart from mathematics, it is worth mentioning that in 1791, Legendre became a member of the committee of the *Académie des Science* to standardize weights and measures. In particular, the committee worked on the metric system and undertook the necessary observations and calculations to compute the length of the metre [53].

In 1824, Legendre refused to vote for the government's candidate for the National Institute and, as a consequence, his pension from *École Militaire*, where he served from 1799 to 1815 as a maths examiner for graduating artillery students, was cut off [52]. This was partially reinstated in 1828 with the change in government, but because of illness and poverty Legendre died a few years later. Shortly, before he died though, he was able to give a proof of Fermat's theorem in the case $n = 5$ [34]. Legendre is not to be forgotten because he has been permanently honoured by having a crater on the moon and a street in Paris named after him. Gustave Eiffel also listed his name, along with 71 other French scientists, on the famous Parisian tower.

Finally, as a point of special interest, all the textbooks, paintings, manuscripts, journal articles, and media sources that have made reference to Legendre over the last two centuries in some way have included the same portrait of him. This has been a picture of him taken from his left side, dressed in formal attire. Recently, in 2009, it was discovered that this portrait was actually of an obscure French politician, Louis Legendre. Thus, the only real portrait of Adrien Legendre is found in 1820 book *Album de 73 Portraits–Charges Aquarelles des Membres de l'Institut*, a book of caricatures of 73 famous mathematicians by the French artist Julien Leopold Boilley [34].

7. Sylvestre François Lacroix (1765-1843)

Sylvestre Lacroix was born in Paris to parents, who were moderately poor but who saw to it that their son would receive a good education, which he did at the *Collège des Quatre Nations*. Lacroix showed significant interest and promise in mathematics at an early age. When he was only 14 years old, he spent considerable time in making lengthy calculations of the motions of planets (later he was to have a lunar crater named after him) [46].

In the following year, in 1780, Lacroix busied himself by taking classes at the *Académie des Sciences* taught by the renowned mathematician Gaspard Monge. These classes covered topics from physics, chemistry, and mathematics (especially geometry since Monge was later to be known as the father of differential geometry). One of Monge's premier works was his *Application de l'analyse à la géométrie,* where he introduced the concept of lines of curvature of a surface in 3-dimensional space. Monge was to have a significant relationship with Napoléon, first accompanying him in 1798 on some of his expeditions in the Mediterranean, and then having Napoléon bestow upon him many honours. Unfortunately, this relationship turned sour when Napoléon started tasting defeat, and by the time Napoléon was defeated at Waterloo, Monge began to fear for his life and was forced to flee France [56].

The friendship between Monge and Lacroix remained strong for most of their lives. Monge, in fact, secured for Lacroix his first position, in

1782, as professor of mathematics at the *École des Gardes de la Marine* at Rochefort [23]. Monge also advised Lacroix to apply himself and conduct research in both partial differential equations and the calculus of variations. By the time Lacroix was 20 years old, he had already submitted two papers to the *Académie des Sciences*, first on partial differences and second on astronomy.

Lacroix's real strength as a mathematician came, not from any new outstanding mathematical results, he might have discovered or any new fields of study he might have developed, but from his role as an educator and an author. In addition to his teaching position at Rochefort, he also taught at the *Lycée* in Paris, and was the mathematics chair at the *École Militaire* in Paris. After the *Militaire* closed, in 1788, he took up a post as professor of mathematics, physics, and chemistry at the *École Royale d'Artillerie* in Besançon. Later, he assisted Monge in teaching at the *École Normale de l'An* III, and shortly thereafter, he became professor of mathematics at the *École Centrale des Quatres Nations*. In 1799, he succeeded to Lagrange's chair at the *École Polytechnique*, a position he would hold for 10 years [23]. From 1805 to 1815, Lacroix taught transcendental mathematics at the *Lycée Bonaparte*, and then became chair of mathematics at both the *Collège de France* and the *Sorbonne*.

Lacroix's main contribution to mathematics, though, was in his writings. His writings on analytic geometry served as models for many later works. It was he who actually proposed the term "analytic geometry", which now accompanies "calculus" as the title on many calculus textbooks and course offerings [23]. He published a three volumes calculus textbook during 1797-1800 titled *Traité du Calcul Différentiel et du Calcul Intégral*. Other works included a ten volumes *Cours des Mathématiques*, *Éléments de Géométrie*, and *Traité des Différences et des Series*. Lacroix was a contributor on some special works such as *Lettres de M. Euler*, *Éléments d'Algèbre par Clairaut*, and *Histoire des Mathématiques*. For more than 50 years, through his writings and lectures, Lacroix contributed to an era of renewal and expansion in the natural and physical sciences and to the training of numerous 19th century mathematicians. His textbooks had a profound influence both in France and beyond. Translations of his books into the English language were used in British universities for nearly half a century. In 1812, the English mathematician and early developer of computing machines, Charles Babbage, set up "*The Analytical Society*" and one of its first acts was to translate into English the more popular 1802 abridged volume *Traité Élémentaire du Calcul Différentiel et du Calcul Intégral*, from which whole generations of students learned their calculus [61].

8. Gabriel Léon Jean Baptiste Lamé (1795-1870)

Gabriel Lamé was born and raised in the town of Tours, located a short distance to the southwest of Paris, and on the lower reaches of the river Loire. During Gallic times, the city was an important crossing point of the Loire. When Tours became part of the Roman Empire during the first century AD, the city was named "Caesarodunum" (hill of Caesar). In the year 732, the famous Battle of Tours was held. Presently, the city is known for its fine wines and the yearly Paris-Tours bicycle race [8].

When Lamé was 18 years old, he moved to Paris and entered the *École Polytechnique*. During his four years, there he published his first research paper *Mémoire sur les Intersections des Lignes et des Surfaces*. Lamé then spent three years studying engineering (his real passion) at the *École des Mines* in Paris, graduating in 1820. During this time, he published his second work on a method to calculate the angles between faces of crystals.

After Lamé graduated from *Mines*, his life took an interesting twist. He and a colleague, B. P. Émile Clapeyron, were lured to Russia (where the academic atmosphere for research was much more attractive than in France) to collaborate with Russian scientists on improving military techniques and industrial development. Alexander I, the Emperor of Russia, was keenly aware of the importance of scientific knowledge and he made a special request to the French government to send him some top quality scientists. Lamé was immediately appointed professor and engineer at an Institute in St. Petersburg, where he lectured on analysis, physics, mechanics, chemistry, and engineering topics. In addition to his teaching, Lamé became involved with building bridges, roads, and railway lines. In fact, when Lamé returned to Paris in 1832, he not only joined an engineering firm, but he was also appointed in chair of physics at *École Polytechnique*. During his engineering consulting work, he was involved with the building of the railways from Paris to Versailles and from Paris to St. Germain [49].

Often problems in engineering led him to study mathematical questions. His work on the stability of vaults and the design of suspension bridges led him to work on elasticity theory. In particular, we encounter the *Lamé constants*, denoted by λ, μ and defined by

$$\sigma_x = 2\mu\varepsilon_{xx} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}), \quad \tau_x = \mu\varepsilon_{xy},$$

where σ and τ are the normal and tangential components of an elastic stress at some point of a linearly-elastic body [30].

Another example is his work on the conduction of heat, which led him to his general theory of curvilinear coordinates. He made particular use of them in transforming *Laplace's equation* into elliptical coordinates, separating the variables, and then solving (the solution is known as a *Lamé function*) the resulting equation [29]. This was also, where we encounter the famous *Lamé curve*, which in two-dimensional rectangular coordinates is the curve given by

$$\left|\frac{x}{a}\right|^m + \left|\frac{y}{b}\right|^m = 1,$$

with $a, b > 0$ and m is any positive rational number. In particular, if $m = 1,$ we get a straight line, if $m = 2,$ we get an ellipse, and $m = 2/3,$ we get an astroid, which is a special case of a hypocycloid. This astroid curve resembles a four-pointed star with concave (inwards-curved) sides. Interesting enough, if we collect three of these stars, colour them yellow, orange, and blue, respectively, and place all three inside a larger circle together with the word "Steelers", we get the logo that appears on the helmets of the Pittsburgh Steelers professional football players [57].

In addition to the Steelers' logo, Lamé curves (frequently known as *superellipses*) have some other useful applications. In 1959, Stockholm, Sweden issued a design challenge for a roundabout in their city square Sergels Torg. The winning proposal came from the Danish scientist and poet Piet Hein (inventor of the game Hex and the Soma cube) and it involved a superellipse with $m = 2.5$ and $a / b = 6 / 5$. The curve was enthusiastically received by the city planners, and was viewed as a comforting blend of elliptical and rectangular beauty. Residents of Stockholm now feel their city possesses an unusual mathematical flavour to rival the catenary curve of St. Louis' Gateway Arch. Hein's ideas were subsequently adopted by Bruno Mathsson, a well-known Swedish furniture designer, who went on to use the superellipse in other practical avenues such as beds, dishes, and tables. By rotating a superellipse around its longest axis, Hein created the *superegg*, a solid egg-like shape that could stand upright on a flat surface, and was subsequently marketed as a novelty toy [17]. Another application finds that the superellipse was used for the shape of the Olympic Stadium in 1968 in Mexico City [62].

Hermann Zapf is a German typeface designer, who has invented more than 200 typefaces, of which some are today among the most widely used standard fonts. Palatino, Optima, Melior, Virtuosa, Aldus, and Kompakt are just some of the typefaces Zapf created early in his career. Melior, in particular, incorporated proportions based on the superellipse [41]. In the early 1990's, Zapf developed a typeface called *AMS – Euler* for the American Mathematical Society. This was a collaborative project with Knuth of Stanford University, who converted Zapf's drawings into digital fonts [65].

In addition to his engineering work, Lamé also contributed to the fields of analysis, differential geometry, and number theory. Regarding this latter area, he discovered a relationship governing the number of divisions required in the Euclidean algorithm for determining the greatest common divisor of two integers [49]. More importantly, though, through his study of the curves $(x / a)^m + (y / b)^m = 1$, he was led to study *Fermat's last theorem*. In 1839, he produced a valid proof that the equation $x^m + y^m = z^m$ has no positive integer solutions when $m = 7$. Then, in March, 1847, at the meeting of the Paris *Académie des Sciences*, Lamé presented his proof that Fermat's equation has no integer solution for any exponent $m > 2$ [6]. Unfortunately, his proof had a flaw in it (factorization of a certain class of integers was not unique), and it would be roughly 150 more years until Andrew Wiles presented an acceptable proof.

Lamé was elected to the *Académie des Sciences* in 1843 and in the following year accepted a position at the Sorbonne in mathematical physics and probability. Many people considered him the leading French mathematician of his time, including the great Carl Gauss, who shared many of the same interests. Lamé's name is one of the 72 French scientists that Gustave Eiffel commemorated on his famous Parisian tower.

9. Joseph Liouville (1809-1882)

Joseph Liouville received his early education in the town of Toul, between Commercy and Nancy, where he was attracted to ancient languages. He then moved to Paris to begin his study of mathematics at the *Collége St. Louis*. Here he became well acquainted with the mathematical research of the day and, in particular, with the journal *Annales des Mathématiques Pures et Appliquées*, ironically the only French mathematics journal. During his studies, he developed a lifelong habit of sketching his mathematical ideas and correspondence in notebooks, of which 340 contain approximately 50,000 pages and are preserved at the *Bibliothèque de l'Institut de France*. This trait would prove of monumental importance later in his life. His early mathematical results and articles dealt with geometry, in which he established some results involving both triangles and quadrilaterals.

In 1825, he entered *École Polytechnique*, which was not surprising since he was the son of a military man; his father, Claude-Joseph, was a captain in Napoléon's army. Besides taking classes in geometry, trigonometry, French, and Latin, his most important course was in analysis and mechanics, the *Cours D'analyse et de Mécanique*, which happened to be alternately taught by the two masters André-Marie Ampère and Augustin Louis Cauchy (who later became known as the father of analysis for his work in both real and complex analysis) [40].

Three years later, as he was working in the field as an engineer, Liouville began having health problems, possibly the early stages of the rheumatism, which racked him later in life. Then, in 1830, he officially resigned from the life of an engineer, and began his career as a full-fledged scientist and mathematician. During the next several years, he authored papers on electrodynamics, an outgrowth of some pioneering work done by Ampère. He showed that Ampère's force law was only one of many possible laws, hence spotting a weakness in Ampère's theory. Other papers included promising work in partial differential equations, an improvement in the mathematical theory of electricity, and substantial contributions in the theory of heat conduction. He had also submitted several papers in pure mathematics involving infinite series and multiple integrals.

As it turned out, though, publication of mathematical papers was starting to become quite difficult. Several journals terminated publication in 1831, including the *Annales des Mathématiques Pures et Appliquées*. By 1833, Liouville began sending his papers, including major papers on fractional calculus and integration in finite terms, to the German journal *Journal für die Reine und Angewandte Mathematik*, but he soon realized the need for additional journals. Consequently, around 1835, Liouville created his own mathematics journal, *Journal des Mathématiques Pures et Appliquées*, which was intended for papers in all fields of pure and applied mathematics, including their histories, and also publishing papers of correspondence between promising partners, and translations of papers in foreign journals [54]. There are those in mathematics, who claim this as Liouville's most important mathematical creation. But it is amazing that a 26 years old could hold the position as editor and a judge of the works of the best mathematicians in the world, while still having the world judge his work! He was clearly well on his way to becoming world famous [40].

During the mid 1830's, Liouville did serious work in celestial mechanics and perturbation theory, specifically with the orbits of some of the major planets. This work was instrumental in getting him elected, in 1839, to the astronomy section of the prestigious *Académie des Sciences*. The following year Liouville was appointed as a professor at the school he had previously attended, the *École Polytechnique*. Here he taught, in conjunction with the Swiss mathematician Jacques Charles François Sturm, the same analysis and mechanics course that Ampère and Cauchy had previously taught. Sturm and Liouville did joint work on boundary value problems of differential equations, and the *Sturm-Liouville problem* and *Sturm-Liouville equations* are standard material in advanced texts. Liouville also has a theorem named after him in complex analysis, which says that any analytic function bounded in the entire plane must necessarily be a constant function. This theorem furnishes short and convenient proofs of some of the most important results in analysis.

For roughly the next 20 years, Liouville was recognized as the premier mathematician in all of Europe. In addition to his continued research into mathematical analysis of many physical problems, let us mention two final footnotes most prominent in his career. The first involves his entrance into the field of algebra, where, among other things, he contributed a new proof of the fundamental theorem of algebra. But, more importantly, he published, in 1846, in his journal the works of Evariste Galois (1811-1832) [63]. Galois' manuscripts had provided the definitive answer to which equations of a given degree admit an algebraic solution; this, in turn, opened up the new mathematical area known as *group theory*, which is now an integral and accepted part of mathematics. Galois had originally submitted his papers to the *Académie des Sciences* in 1829, but the papers were either lost, misplaced, or rejected, first by Augustine-Louis Cauchy, then by Joseph Fourier, and then by Siméon-Denis Poisson. Finally, he sent a summary of his notes to a friend. Shortly, thereafter he was killed in a duel, and it was not until 1843 that the notes made their way to Liouville.

The second footnote describes some of Liouville's work in the field of number theory. Here he demonstrated that if the equation $x^n + y^n = z^n$ has no solution in integers, then neither does $x^{2n} + y^{2n} = z^{2n}$. He also gave a new derivation of the law of quadratic reciprocity, investigated properties of the Euler phi-function, and studied relationships involving the divisors of integers. The Liouville λ-function was find out of this latter investigation. In 1844, he proved that neither e (the second most famous number in higher mathematics) nor e^2 can be a root of a quadratic equation with rational coefficients. This was a step in the marvelous chain of arguments that led from Lambert's proof in 1761 that π is irrational to Hermite's proof that e is transcendental (1873) and the final proof by Lindemann in 1882 that π is transcendental. Now, in closing, we need to discuss this term "transcendental".

All real numbers can be split into two disjoint and mutually exclusive sets, known as the *rational numbers* and the *irrational numbers*. The rationals would include such numbers as $7, -3, \frac{17}{5}$. School children know these numbers as the whole numbers and the fractions. The irrational numbers include everything else, such as radicals, like $\sqrt{2}, \sqrt[3]{10}$, and other numbers such as log(5) and sin(1). But there is another way to partition the real numbers that is important in mathematics, and that is with the algebraic numbers and the nonalgebraic numbers. The algebraic numbers are all the numbers that can be roots of polynomial equations, where the polynomial has integer coefficients. Consequently, both $\frac{17}{5}$ and $\sqrt[3]{10}$ are algebraic because they are the roots of $17x - 5 = 0$ and $x^3 - 10 = 0$, respectively. Clearly, there are infinitely many of these algebraic numbers. The nonalgebraic numbers, henceforth known as *transcendental numbers*, comprise whatever numbers remain after deleting the algebraic numbers from all of the reals. Are there really any numbers at all in this class? This brings us to the result that Liouville is most famous for, because he showed that there were numbers in the transcendental class! In particular, he showed that transcendental numbers could be approximated quite closely by ratios of integers p/q, whereas algebraic numbers could not unless the denominator q was quite large [10]. Liouville demonstrated this result in 1840 by exhibiting the number (known as Liouville's number)

$$L = 0.110001000000000000000001000\ldots,$$

where the only nonzero digits are the 1's located in the 1st, 2nd, 6th, $\ldots$, $(n!)$th places [9]. Then in 1844, he showed that there were an infinite number of such transcendental numbers, but he was unable to show that e was transcendental, which was what he really wanted to do. It was left to Georg Cantor, some 25 years later, to show that the set of transcendental numbers was in fact "much larger" than the set of algebraic numbers, even though both sets are infinite.

When Liouville ended his reign as editor of his *journal* in 1874 (it was soon to become known as the *Journal de Liouville*), it also pretty much signaled the end of his mathematical activity. His exhaustive list of publications numbers well over 400. His research in both pure and applied mathematics together with his teachings and journal publications had a most fruitful influence on French mathematics.

10. Edmond Nicolas Laguerre (1834-1886)

Edmond Laguerre was born in Bar-le-Duc, in the northeast quadrant of France. Not enough is known about his family, or his early days. He apparently suffered from some health problems, which caused his parents to move him from one public school to another. His scholarship in school was good enough, though, to gain him admittance in 1852 to the prestigious *École Polytechnique* in Paris. The subjects he excelled in were languages and mathematics, but his overall showing in class was not especially good due, in some part, to his health issues.

His first mathematical love was the area of geometry, in which he would continue to do brilliant work for most of his life. Among his works are papers on foci of algebraic curves, on geometric forms that are transformed into themselves by inversions, on fourth-order curves, and studies of curvature and geodesics. While still a student at *École*, he published an important paper on the theory of foci, that investigated the angle between lines in the complex projective plane [48].

Two years later, after leaving school, Laguerre naturally entered the military and accepted a commission as an artillery officer. He would maintain this position for an entire decade while leaving his mathematical studies on the "back burner". In 1864, Laguerre resigned his commission and returned to *École Polytechnique* to become a mathematics tutor. He would remain there for the rest of his life, serving in various other capacities. Concurrent with employment at *École,* Laguerre was appointed in 1883 as the chair of mathematical physics at the *Collège de France*. Three years later his health took a turn for the worse, so he returned home to Bar-le-Duc, and soon thereafter died.

Laguerre did not have the lofty mathematical status of some of his fellow countrymen, but he was nevertheless a most competent mathematician with roughly 140 published papers. His name appears in textbooks in a few places.

One of the most basic and fundamental results in algebra is that a real polynomial of degree n has at most n real zeros. Rene Descartes' *rule of signs* says, in addition, the number of positive zeros is no more than the number of sign changes in the sequence of coefficients. This used to be a standard result presented in high school, but appears to have fallen by the wayside. Laguerre was able to derive a similar statement about a more general class of functions than polynomials [24].

In statistics, the *Laguerre-Samuelson inequality* states that every element in the data set $\{x_1, x_2, \cdots, x_n\}$ is within $\sqrt{n-1}$ standard deviations of the mean $\bar{x}$. Symbolically, this says

$$\bar{x} - s\sqrt{n-1} \le x_i \le \bar{x} + s\sqrt{n-1},$$

where s is the standard deviation of the data set, and for all choices of $i = 1, 2, \cdots, n$ [27]. The inequality is not particularly useful since the bounds are not very tight. Incidentally, the name Samuelson refers to the American economist Paul A. Samuelson (1915-2009), who was a professor of Economics at Massachusetts Institute of Technology, and was the first American to win (in 1970) the Nobel Prize in Economics [59].

One of the essential ingredients in numerical analysis is the employment of an algorithm to locate the zeros of a function, that is, to numerically solve the equation $f(x) = 0$. One of the most common methods is *Newton's method*. Hidden in the literature, though, is *Laguerre's method*, which is designed to find the zeros of polynomials. It is a relatively easy 3-4 steps procedure that is akin to *Halley's method* (of Halley's comet fame) [28].

Finally, it is in the field of analysis, where Laguerre's name really stands out. There is a special class of functions, known as the *Laguerre polynomials*, defined by the n-fold derivative (a Rodrigues formula)

$$L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}(x^n e^{-x}), \quad n = 0, 1, 2, \ldots,$$

or given by the equivalent definition

$$L_n(x) = \sum_{k=0}^{n} \frac{(-1)^k n!}{(k!)^2 (n-k)!} x^k.$$

These polynomials satisfy *Laguerre's differential equation* [12]

$$xy'' + (1-x)y' + ny = 0.$$

The polynomials $L_n(x)$ are important in the quantum mechanics of the hydrogen atom, and as such, were used by the physicist Erwin Schrödinger in 1926. They satisfy the important orthonormality relationship

$$\int_0^\infty e^{-x} L_n(x) L_k(x)\,dx = \begin{cases} 0 & k \ne n, \\ 1 & k = n. \end{cases}$$

Laguerre came across them, though, in his study of the improper integral $\int_x^{\infty} \frac{e^{-u}}{u}\,du$. He was able to express the integral as a non-simple infinite continued fraction, one of the earliest such examples that was convergent [48].

As brilliant as Laguerre was, many writers tend to agree that Laguerre spent too much time working on small details. Had he put pieces together into a larger single theory, then he would have achieved more fame and lasting recognition [48].

11. François Édouard Anatole Lucas (1842-1891)

Édouard Lucas was born and raised in the town of Amiens, located in the north central portion of France. He received his formal schooling at the *École Normale* in Amiens, with the intention of becoming a mathematics educator. A gifted student and researcher, he quickly found employment after graduation at the Paris Observatory, where he worked as an assistant under the directorship of Urbain Le Verrier. Le Verrier was appointed to teach astronomy at *École Polytechnique* in 1837, but spent the majority of his life running the Paris Observatory. His claim to fame is that his calculations of the irregularities in Uranus' orbit led to the discovery of Neptune. With the advent of the Franco-Prussian war (1870-71), Lucas was called into duty, where he was commissioned as an artillery officer. When the war ended, with France on the losing side, Lucas returned to his true love of mathematics and became a professor of mathematics, first at *Lycée Saint Louis*, and then at *Lycée Charlemagne*, both schools located in Paris [39]. Lucas gained a reputation at both for being an accessible and entertaining teacher.

Lucas' mathematical career can aptly be divided into two parts, one dealing with number theory and the other part with recreational mathematics. This latter is quite unusual since very few mathematicians (then and now) busy themselves with this area.

It is in number theory, where Lucas is most prominently known. We begin with undoubtedly the most famous sequence, the *Fibonacci sequence*, of numbers in mathematics, denoted by $0, 1, 1, 2, 3, 5, 8, \ldots,$ where each term, starting with the third term, is the sum of the two previous terms. Symbolically, this sequence is defined recursively by $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$, for all $n \geq 2$. The Fibonacci sequence is named after Leonardo of Pisa, who was known, in fact, as Fibonacci. Fibonacci's book *Liber Abaci* in 1202 was where the famous sequence was introduced to Western European mathematics. This sequence of numbers seems to have an inexhaustible supply of applications, as evidenced by the journal "*The Fibonacci Quarterly*", which has been in print for almost 50 years. Lucas is known for his study of the Fibonacci sequence; in fact, he is credited with naming it such as [25]. A related sequence, known as the *Lucas sequence*, contains the numbers $2, 1, 3, 4, 7, 11, \ldots$ and is defined, similar to the Fibonacci sequence, by $L_0 = 2$, $L_1 = 1$, and $L_n = L_{n-1} + L_{n-2}$, for all $n \geq 2$ [20]. Several interesting identities that Lucas deduced were

(i) $F_0 + F_1 + F_2 + \cdots + F_n = F_{n+2} - 1,$

(ii) $L_n = F_{n-1} + F_{n+1},$

(iii) $L_n = F_{n+2} - F_{n-2}.$

Many more identities can be found in the 1969 book *Fibonacci and Lucas Numbers* by Verner E. Hoggart, Jr..

There is an explicit formula for the n-th Fibonacci number, namely,

$$F_n = \frac{\alpha^n}{\sqrt{5}} - \frac{\beta^n}{\sqrt{5}}, \text{ where } \alpha = \frac{1+\sqrt{5}}{2} \text{ and } \beta = \frac{1-\sqrt{5}}{2}.$$

Lucas claimed to have discovered this relationship, although most writers give the credit to Jacques Philippe Marie Binet (although Abraham de Moivre knew it a century earlier), who deduced it in 1843, when Lucas was one year old [11]. Incidentally, the Lucas numbers can be explicitly given by $L_n = \alpha^n + \beta^n$ [22].

Another area of number theory research by Lucas involved prime numbers, the so-called "building blocks" of the integers. In 1876, after 19 years of testing, Lucas was able to verify that the number $2^{127}-1$ was prime [27]. He also verified that $2^{67}-1$ was not prime, and he did this without exhibiting any factors of the number. In both cases, these numbers have the general form 2^n-1, which classifies them as *Mersenne numbers*, M_n, named after Marin Mersenne (1588-1648). These numbers figure prominently in the determination of perfect numbers, which at one time held a special mystic appeal to many people. The most interesting Mersenne numbers are those that are prime, in which case the exponent on 2 must be a prime, say *p*. Thus, the question was, for which *p* is 2^p-1 also a prime. Mersenne, a monk by trade, but an amateur mathematician on the side, had stated that 2^p-1 would be a prime when *p* assumed the values $p = 2, 3, 5, 7, 13, 17, 19, 31, 67, 127,$ $257,$ and for no other *p* below 257. Lucas showed, therefore, that Mersenne's list was incorrect because it should not have included $p = 67$. Mersenne also erred by omitting $p = 61, 89,$ and 107. The number $2^{127}-1$ happens to be a 39-digit number, and it held the record for being the largest known prime for nearly 70 years [9]; roughly 31 Mersenne primes have been found since then, with the 12 million digit $M_{43112609}$ the largest. It still does have the distinction of being the largest prime established without any computer or machine assistance.

There are special techniques for establishing the primality of numbers of the form 2^p-1, which happens to be the form for practically all the new primes that are discovered each year by high powered computers. One test, used by Lucas, and later refined by the American mathematician Derrick H. Lehmer (1905-1991), is the *Lucas-Lehmer test*, which says that if *p* is an odd prime, then 2^p-1 is prime, if and only if 2^p-1 divides S_{p-1}, where this sequence is defined recursively by $S_1 = 4$ and $S_{n+1} = S_n^2 - 2$ for all $n > 1$ [55].

Lucas' other love was recreational mathematics, and his 4 volume book *Récréations Mathématiques*, published in 1894, is considered a classic. His most famous recreation, though, is the *Tower of Hanoi* puzzle that he introduced in 1883 under the name M. Claus (a permutation of Lucas) [55]. This puzzle consists of three vertical pegs and an assortment of circular disks situated on one of the pegs. The object is to move all the disks onto one of the other pegs, following several simple rules [39]. This is a common exercise in computer programming classes to illustrate the concept of recursion.

Some other puzzles that he introduced or worked on included magic squares, disentanglement puzzles (also known as *Chinese rings*, or the *Devil's needle*), and the ménage problem. This latter problem consists of a group of married couples, who need to cross a river in a small boat, subject to certain restrictions. In 1889, Lucas published his Dots and Boxes game, which is a pencil and paper game played by two people [13]. In the same year, Lucas won a gold medal for his puzzles at the World's Fair in Paris.

One last game worth mentioning is his famous *cannonball problem*, also known as the *square pyramid puzzle* [27]. Here he challenged his readers in 1875 to determine whether it was possible for a pyramidal stack of cannonballs to be rearranged into the shape of a square, and if so, what were all the possible ways this could happen. Algebraically, this meant to find all the possible solutions to

$$1^2 + 2^2 + 3^2 + \cdots + x^2 = y^2,$$

where x and y are positive integers. Lucas knew one solution was $x = 24$ and $y = 70$, and he thought he had a proof that showed this was the only solution. Unfortunately, his proof had a flaw in it. It was not until 1918 that Watson proved (by using high powered mathematical machinery) that this was the only solution. Since then simpler solutions have been obtained, notably in 1985 by De Gang Ma and 1990 by Anglin [1]. It is not uncommon to find golf balls stacked in this pyramidal shape on the driving range at many of today's well-to-do courses.

Lucas was attending a banquet in 1891 when a waiter accidentally dropped a plate and a chard flew up and struck Lucas in the face. He contracted blood poisoning from the incident and died a few days later [38].

12. Henri Léon Lebesgue (1875-1941)

Henri Lebesgue, the most recent of our special group of mathematicians, was born in Beauvais in northern France of middle-class parents. His father was a printer/typesetter and his mother was an elementary school teacher. Unfortunately, the father died of tuberculosis when Lebesgue was young, and Lebesgue himself suffered from poor health all of his life. Lebesgue's early studies were at the local *Collège de Beauvais*, and then he moved to Paris to study at *Lycée Saint Louis* and *Lycée Louis-de-Grand*. At the age of 19 he entered the *École Normale Supérieure*, and received his teaching diploma in mathematics three years later in 1897. He graduated third in his class, which is pretty remarkable considering that he had a penchant for not applying himself in classes that he did not favour. One cute story has him, passing his chemistry class (not one of his favourite classes) by deliberately mumbling his answers to the examiner, who was hard of hearing [32]!

After graduating in 1897, Lebesgue spent two years working in the school library before gaining a full time teaching position at the *Lycée Central* in Nancy, where he would stay for three years. But it is during this five years period that he conducted some of his most important work with regard to the integral calculus–work that would make his name forever famous.

The theory of integration has a long and colourful background. It actually corresponds to finding the area under the graph of a function. Finding areas and volumes of regions dates back to the third century BC with work done by Archimedes. Two thousand years later, Newton and Leibniz independently discovered that integration was a sort of opposite to differentiation and, hence, a large class of integrals could be computed.

During the following century, Cauchy developed a theory of *limits* and *convergence* that provided much needed rigor to the calculus. The German Bernhard Riemann followed up on this by formalizing his definition of the *Riemann integral*, a standard feature now in textbooks. The Riemann integral did have its limitations though, especially, with regard to discontinuous functions.

While Lebesgue was working in the library at *École Normale*, he read the works of Rene Baire (also a recent graduate of *École Normale*) pertaining to discontinuous functions [51]. He also read what Émile Borel had done regarding a theory of measure that generalized the concept of area to new types of regions obtained via limits. Lebesgue thus put two-and-two together in his doctoral thesis, *Intégrale, longueur, aire*, at the Sorbonne. This thesis extended Borel's theory of measure, now to be known as the *Lebesgue measure*, and defined his new integral, the *Lebesgue integral*, geometrically and analytically, and established nearly all the basic properties of integration. In an obituary of Lebesgue from the *Journal of the London Mathematical Society* are the words "It cannot be doubted that this dissertation is one of the finest, which any mathematician has ever written".

In addition to his teaching position at Nancy, Lebesgue was to hold other university positions at Rennes (1902-1906), at Poitiers (1906-1910), at the Sorbonne (1910-1919), and at the *Collège de France* (1921-1941). Twice (1903, 1905) he was invited to give the Cours Peccot lecture at the *Collège de France*, and in 1922, he was elected to the *Académie des Sciences* [51]. He was also elected to the Royal Society, the Royal Academy of Science of Belgium, the Academy of Bologna, the Royal Dannish Academy of Sciences, the Romanian Academy, and the Kraków Academy of Science and Letters.

Lebesgue wrote two books, *Leçons sur L'intégration et la Recherche des Functions Primitives* (1904), and *Leçons sur les Séries Trigonométriques* (1906), and roughly 90 papers on topics ranging from measure theory, integration, topology, geometry, history of mathematics, potential theory, set theory, the calculus of variations, the theory of surface area, and number theory. In this latter area, Lebesgue worked on the famous Dirichlet problem, which says if integers a, b are relatively prime, then the arithmetic sequence $a + kb$ contains infinitely many primes. Lebesgue also proved the interesting result that every positive integer can be expressed as the sum of a square and two triangular numbers $(41 = 4^2 + [10 + 15])$ [6].

During his later years, Lebesgue's writings were concerned with pedagogical issues and elementary geometry. In his honour, a lunar crater has been named after him.

This paper does not exhaust all the well known French mathematicians of the 18-th century, whose last name begins with "L". Pierre Laurent, Émile Lemoine, and Émile Léger are three, who have been omitted simply because this author had to draw the line somewhere. Similarly, Pierre-Louis Lions and Laurent Lafforgue are omitted for they are famous French mathematician still living today, and both have won the prestigious Fields Medal (the mathematician's analogy to a Nobel Prize) in 1994 and 2002, respectively. But the historic journey presented by these 11 mathematicians herein is as exciting and colourful as it gets, and it deserves full recognition when discussing both the history of France and the history of mathematics.

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